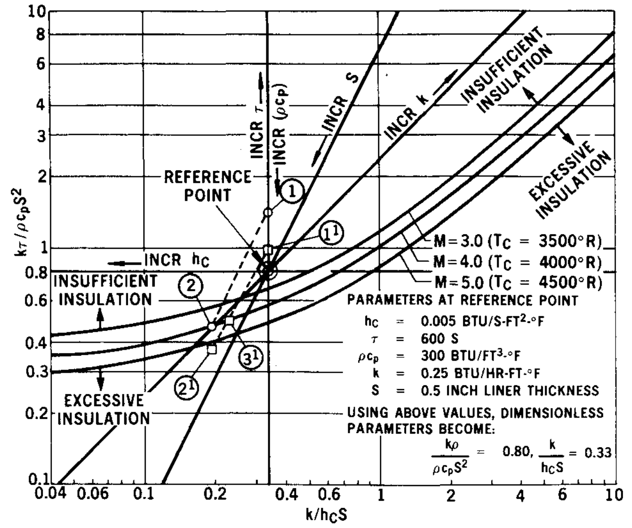

Fig. 5 Required insulation thickness for embedded combustor case.

tain this solution, the same input is required as for the other configuration with the addition of τ, $\rho C_p|_{liner}$, and $\rho C_p|_{case}$; and the exception of ϵ and h_o.

Figure 5 is the parametric solution to this problem. For any given insulation thickness and flight environment, the coordinates of a point in the figure can be computed, either with or without $\rho C_p|_{case}$. This point will fall above the Mach number curve if the insulation is insufficient, and below it if excessive insulation is present. For singular parameter changes, the point moves in the direction indicated by lines passing through the reference point marked "← incr S", "incr k →," etc.

To illustrate this solution, consider the same example as before with $\tau = 525$ sec, $\rho C_p|_{liner}$ = 30 BTU/ft³ -°F, $\rho C_p|_{case}$ = 52 BTU/ft³ °F, and $S_{case} = 0.125$ in. Neglecting the case and taking $S_{liner} = 0.5$ in. as the initial guess, point (1) in Fig. 5 becomes $(k\tau/\rho C_p S^2, k/h_c S)$ = (1.40, 0.333). A line through this point parallel to the "incr S" line is constructed; it intersects the $M=4$ curve at (0.465, 0.195) and is labeled (2). The required thickness is

$$S = 0.5[(k/h_c s)|_{(1)}/(k/h_c S)|_{(2)}] =$$

$$0.5\ (0.333/0.195) = 0.85 \text{ in.}$$

If the effect of the case is included, the same general procedure is followed, except that the heat capacity term now varies with liner thickness according to the relation

$$(\rho C_p)_{eff} = (\rho C_p)_{liner} + \frac{(\rho C_p S)_{case}}{S_{liner}}$$

The points on the plot corresponding to a liner thickness of 0.5 in. [marked (1′)] and 0.85 in. [marked (2′)] are calculated

For (1′):

$$(\rho C_p)_{eff} = 30 + \frac{52(0.125)}{0.5} = 43$$

$$\frac{k\tau}{\rho C_p S^2} = 1.4\ (30/43) = 0.976$$

$$\left[\frac{k\tau}{\rho C_p S^2}, \frac{k}{h_c S}\right]_{(1')} = (0.976,\ 0.333)$$

Similarly for (2′):

$$\left[\frac{k\tau}{\rho C_p S^2}, \frac{k}{h_c S}\right]_{(2')} = (0.371,\ 0.195)$$

Lines parallel to the "incr S" line are drawn through these points to the $M=4$ curve. The solution is between these intercepts in approximate proportion to the lengths of the constructed lines. Let $k/h_c S = 0.230$ be considered as one solution coordinate. The corresponding liner thickness is $S = 0.5$ (0.333/0.230) = 0.72 in. The other coordinate is now calculated, $k\tau/\rho C_{p_{eff}} S^2$ = 0.490 which happens to be the other coordinate of the $M=4$ curve at $k/h_c S = 0.230$. This, therefore, is the correct solution point marked (3′) in the figure. In this example accounting for the heat capacity of the motor case allows a 15% reduction in liner thickness.

Response of a Dual-Spin Spacecraft with Flexible Appendages via Modal Analysis

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Nomenclature

m	$= n \times n$ constant, symmetric inertia matrix for the spacecraft
g	$= n \times n$ constant, skew symmetric gyroscopic matrix for the spacecraft due to Coriolis forces
k	$= n \times n$ constant, symmetric stiffness matrix for the spacecraft due to elastic restoring forces and centrifugal forces
$q(t)$	$= n$-dimensional configuration vector (including rotational and elastic motion)
$f(t)$	$= n$-dimensional generalized force vector
M	$= 2n \times 2n$ "inertia matrix" for the spacecraft
G	$= 2n \times 2n$ "gyroscopic matrix" for the spacecraft
$x(t)$	$= 2n$-dimensional state vector
$X(t)$	$= 2n$-dimensional generalized force vector [associated with $x(t)$]
ω_r	= natural frequency of oscillation of the spacecraft
y_r, z_r	= natural modes belonging to ω_r
I	= moment of inertia of entire spacecraft in the nominal configuration about a transverse axis
J	= moment of inertia of the rotor R
Ω	= spin velocity of the rotor
θ_1, θ_2	= attitude angles of the despun section
μ	= single mass simulating the simplified appendage
r_3	= distance from the system mass center to μ
u_1, u_2	= elastic displacements of μ
σ_o	= natural frequency of simplified appendage
v_1, v_2	= elastic velocities of μ
Ω_1, Ω_2	= nutational velocities
$\hat{M}$	= moment impulse

Introduction

EARLY spacecraft were relatively small in size and could be idealized as rigid bodies. In certain cases, the mission required that the spacecraft maintain a given orientation in space, a requirement that could often be met by means of spin stabilization. However, energy

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dissipation restricted spin stabilization to the axis of "greatest moment of inertia." The idea of a "dual-spin" spacecraft represents a natural extension of the single-spin rigid body, motivated by the desire to stabilize a spacecraft consisting of a despun section and a spinning section about the axis of minimum moment of inertia. The term dual spin refers to a spacecraft consisting of two main bodies rotating relative to one another about a common axis, the spin axis. In earlier applications the dual-spin spacecraft consisted of a despun platform and a rotor, as well as dampers idealized as linear oscillators.[1-3] More recent dual-spin spacecraft include flexible members, such as solar panels and antennas.[4] Quite often, but not exclusively, the flexible appendages are attached to the nonspinning section.

The inclusion of flexible appendages introduced a significant complication in the dynamic analysis of spacecraft because of the relatively large number of degrees of freedom required for an adequate simulation. For small motions about the uniform spin equilibrium the equations of motion of a symmetric spacecraft resemble those of a linear gyroscopic system. Quite recently, a modal analysis for the response of linear gyroscopic systems[5,6] has been developed. The method is especially designed for systems with a large number of degrees of freedom. The method is used here to obtain closed-form response of an undamped dual-spin spacecraft with flexible appendages.

The equations for the small motions of a typical dual-spin spacecraft with flexible appendages and negligible damping can be written, to the first approximation, in the form[4]

$$m\ddot{q}(t) + g\dot{q}(t) + kq(t) = f(t) \qquad (1)$$

Equation (1) describes the motion of an undamped linear gyroscopic system and lends itself to closed-form solution by the modal analysis of Refs. 5 and 6. The method works with the state space instead of the configuration space. Indeed, the n second-order equations (1) are transformed into $2n$ first-order equations having the matrix form

$$M\dot{x}(t) + Gx(t) = X(t) \qquad (2)$$

where

$$x(t) = [\dot{q}^T(t) \mid q^T(t)]^T \qquad X(t) = [f^T(t) \mid 0^T]^T \qquad (3)$$

are the $2n$-dimensional state vector and associated generalized force vector, respectively, and

$$M = \left[\begin{array}{c|c} m & 0 \\ \hline 0 & k \end{array}\right], \quad G = \left[\begin{array}{c|c} g & k \\ \hline -k & 0 \end{array}\right] \qquad (4)$$

are $2n \times 2n$ "inertia and gyroscopic matrices," respectively. Note that the first is symmetric and the second is skew symmetric. It is shown in Ref. 6 that the general solution of Eq. (2) is

$$x(t) = \sum_{r=1}^{n} \{ \int_0^t [(y_r y_r^T + z_r z_r^T) X(\tau) \cos \omega_r (t-\tau)$$
$$+ (y_r z_r^T - z_r y_r^T) X(\tau) \sin \omega_r (t-\tau)] \mathrm{d}\tau$$
$$+ (y_r y_r^T + z_r z_r^T) Mx(0) \cos \omega_r t$$
$$+ (y_r z_r^T - z_r y_r^T) Mx(0) \sin \omega_r t \} \qquad (5)$$

where $x(0)$ is the initial state vector and ω_r^2, y_r are solutions of the eigenvalue problem associated with Eq. (2), i.e., they satisfy

$$\omega_r^2 M y_r = K y_r, \quad \omega_r^2 M z_r = K z_r \qquad (6)$$

in which

$$K = G^T I^{-1} G \qquad (7)$$

is a $2n \times 2n$ symmetric matrix. The eigenvalue problem (6) was discussed in great detail in Ref. 5.

Response of an Axisymmetric Dual-Spin Spacecraft with a Flexible Appendage

Reference 4 shows that a simple mathematical model which retains many of the dynamical characteristics of a more complicated dual-spin spacecraft with flexible appendages attached to the despun section can be taken in the form of "a rigid axisymmetric rotor attached to a massless platform with an appendage consisting of a particle of mass μ suspended on a massless elastic cantilever projecting along the extension of the rotor spin axis." Such a mathematical model is shown in Fig. 1. The differential equations for the free oscillation of this system are given in Ref. 4 in the form

$$I\ddot{\theta}_1 + J\Omega\dot{\theta}_2 - \mu r_3 \ddot{u}_2 = 0, \qquad I\ddot{\theta}_2 - J\Omega\dot{\theta}_1 + \mu r_3 \ddot{u}_1 = 0$$
$$r_3\ddot{\theta}_2 + \ddot{u}_1 + \sigma_0^2 u_1 = 0, \qquad -r_3\ddot{\theta}_1 + \ddot{u}_2 + \sigma_0^2 u_2 = 0 \qquad (8)$$

where all the quantities are as defined in the Nomenclature.

Let us consider the response of the system (8) to an impulsive torque applied about axis 1. Introducing the notation $\dot{\theta}_i = \Omega_i \, (i=1,2)$ as well as the auxiliary variables

$$v_i = \dot{u}_i, \qquad i = 1,2 \qquad (9)$$

and considering the state vector

$$x = [u_1 \; u_2 \; v_1 \; v_2 \; \Omega_1 \; \Omega_2]^T \qquad (10)$$

Eqs. (8) and (9) can be written in the form (2) in which

$$M = \begin{bmatrix} \mu\sigma_0^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu\sigma_0^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 & \mu r_3 \\ 0 & 0 & 0 & \mu & -\mu r_3 & 0 \\ 0 & 0 & 0 & -\mu r_3 & I & 0 \\ 0 & 0 & \mu r_3 & 0 & 0 & I \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & -\mu\sigma_0^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu\sigma_0^2 & 0 & 0 \\ \mu\sigma_0^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu\sigma_0^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J\Omega \\ 0 & 0 & 0 & 0 & -J\Omega & 0 \end{bmatrix} \qquad (11)$$

and it is clear that M is symmetric and G is skew symmetric. Moreover, M is positive definite, so that the theory of Refs. 5 and 6 is applicable. Denoting the moment about axis 1 by $M(t) = \hat{M}\delta(t)$, where $\hat{M}$ is a moment impulse and $\delta(t)$ is the Dirac delta function, the generalized force vector becomes

$$X = \hat{M}(t) \; [0 \; 0 \; 0 \; 0 \; 1 \; 0]^T \qquad (12)$$

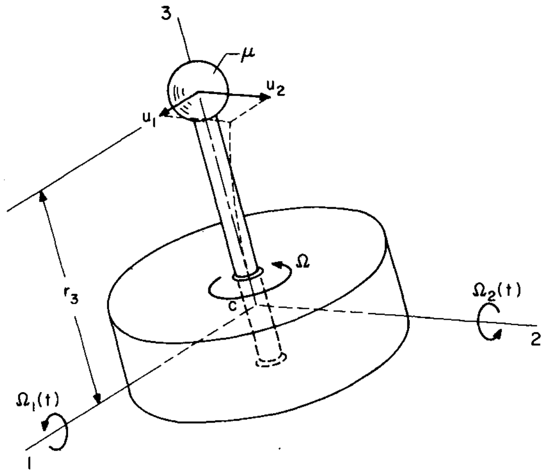

Fig. 1 Axisymmetric dual-spin spacecraft.

Introducing Eqs. (11) into Eqs. (6) and (7) and solving the eigenvalue problem, we obtain the natural frequencies ω_r and the natural modes y_r, $z_r (r=1, 2, 3)$. Then, inserting Eq. (12) in conjunction with the eigenvalue problem solution into Eq. (5), we can write the response in the form

$$x(t) = \hat{M} \sum_{r=1}^{3} [(y_{r5} y_r + z_{r5} z_r) \cos\omega_r t + (y_{r5} z_r - z_{r5} y_r) \sin\omega_r t] \tag{13}$$

where y_{r5} and z_{r5} are the fifth components of the vectors y_r and z_r, respectively.

It should be further noted that the matrices M and G are not only symmetric and skew symmetric, respectively, but in addition they possess special structures, which is a reflection of the fact that the system is axially symmetric. As a result, the eigenvalue problem can be reduced by a factor of two.[7] Indeed, introducing the 2×2 matrices

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{14}$$

where 1 is the unit matrix of order two and i is the matrix counterpart of the imaginary number $i=\sqrt{-1}$, the matrices M and G can be rewritten in the partitioned forms

$$M = \begin{bmatrix} (\mu\sigma_0^2)I & 0 & 0 \\ 0 & (\mu)I & (-\mu r_3)i \\ 0 & (\mu r_3)i & (I)I \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & (-\mu\sigma_0^2)I & 0 \\ (\mu\sigma_0^2)I & 0 & 0 \\ 0 & 0 & (-J\Omega)i \end{bmatrix} \tag{15}$$

which can be operated with as if they were 3×3 matrices, as shown in Ref. 7.

Numerical Results

The preceding formulation was applied to the response of the spacecraft. The system parameters were chosen as

$$\sigma_0 = 4\pi \text{ rad sec}^{-1}, \quad \Omega = 2\pi \text{ rad sec}^{-1}, \quad J = 360 \text{ slug ft}^2$$

$$I = 1{,}800 \text{ slug ft}^2, \quad \mu = 30 \text{ slug}, \quad r_3 = 2\sqrt{10} \text{ ft} \tag{16}$$

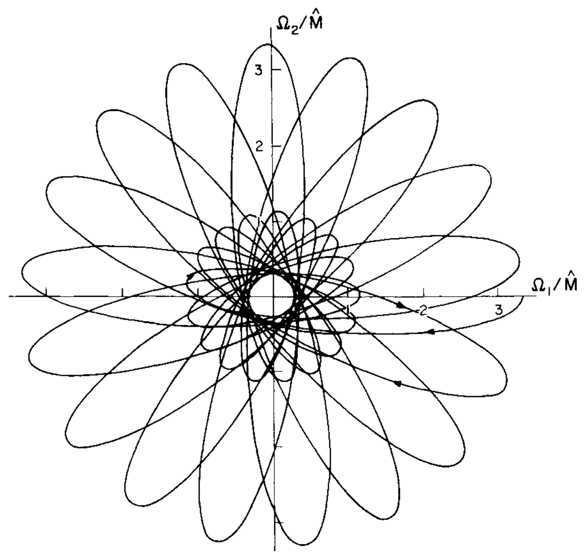

Fig. 2 System response. Scale: 1 in. $= 10^{-3}$ rad lb^{-1} ft^{-1} sec^{-2}.

Using the method of Ref. 7, the solution of the eigenvalue problem was obtained in the form

$$\omega_1 = 1.248342 \text{ rad sec}^{-1}$$

$$y_1 = 10^{-2}[0.117842 \quad 0 \quad 0 \quad 0.147107 \quad -2.333720 \quad 0]^T \tag{17a}$$

$$z_1 = 10^{-2}[0 \quad -0.117842 \quad 0.147107 \quad 0 \quad 0 \quad 2.333720]^T$$

$$\omega_2 = 20.613364 \text{ rad sec}^{-1}$$

$$y_2 = 10^{-1}[0 \quad 0.105324 \quad 2.171080 \quad 0 \quad 0 \quad -0.215702]^T \tag{17b}$$

$$z_2 = 10^{-1}[-0.105324 \quad 0 \quad 0 \quad 2.171080 \quad 0.215702 \quad 0]^T$$

$$\omega_3 = 23.134933 \text{ rad sec}^{-1}$$

$$y_3 = 10^{-1}[0.099381 \quad 0 \quad 0 \quad 2.299170 \quad 0.256274 \quad 0]^T \tag{17c}$$

$$z_3 = 10^{-1}[0 \quad -0.099381 \quad 2.299170 \quad 0 \quad 0 \quad -0256274]^T$$

These results compare quite well with the results of Ref. 4, in which the natural frequencies of the system were obtained by solving the characteristic equation. Indeed, the results of Ref. 4 are: $\omega_1 = 1.260$ rad sec^{-1}, $\omega_2 = 20.571$ rad sec^{-1}, $\omega_3 = 23.135$ rad sec^{-1}. The small difference in the results may be attributed to the inaccuracy inherent in the graphical solution of the characteristic equation used in Ref. 4.

The response of the system is obtained in closed form by inserting Eqs. (17) into Eq. (13). The response is displayed in Fig. 2 in the form of the plot $\Omega_2(t)/\hat{M}$ vs $\Omega_1(t)/\hat{M}$, and it is recognized as being typical of the response of a gyroscopic system (see Ref. 6, Fig. 2).

Conclusions

An axisymmetric dual-spin spacecraft with flexible appendages when perturbed slightly from the uniform spin equilibrium behaves like a linear gyroscopic system. Hence, its response can be obtained in closed form by the modal analysis of Refs. 5 and 6.

References

[1] Likins, P.W., "Attitude Stability Criteria for Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets,* Vol. 4, April 1967, pp. 1638-1643.

[2] Mingori, D.L., "Effects of Energy Dissipation on the Attitude Stability of Dual-Spin Satellites," *AIAA Journal,* Vol. 7, Jan. 1969, pp. 20-26.

[3] Pringle, R., Jr., "Stability of the Force-Free Motions of a Dual-Spin Spacecraft," *AIAA Journal,* Vol. 7, June 1969, pp. 1054-1063.

[4] Gale, A.H. and Likins, P.W., "Influence of Flexible Appendages on Dual-Spin Spacecraft Dynamics and Control," *Journal of Spacecraft and Rockets,* Vol. 7, Sept. 1970, pp. 1049-1056.

[5] Meirovitch, L., "A New Method of Solution of the Eigenvalue Problem for Gyroscopic Systems," *AIAA Journal,* Vol. 12, Oct. 1974, pp. 1337-1342.

[6] Meirovitch, L., "A New Modal Method for the Response of Structures Rotating in Space," presented as paper 74-002 at the 25th International Aeronautical Congress of the I.A.F., Amsterdam, The Netherlands, Sept. 30-Oct. 5, 1974, to be published in *Acta Astronautica.*

[7] Meirovitch, L., "On the Reduction of the Eigenvalue Problem for Spinning Axisymmetric Structures," AIAA Paper 75-159, Pasadena, Calif., Jan. 1975, to be published in *AIAA Journal.*

Errata

Large Space Telescope Oscillations Induced by CMG Friction

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[JSR **12,** 96-105 (1975)]

IN the article, the captions for Figs. 2 and 4 are reversed.

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Low Reynolds Number Effect on Hypersonic Lifting Body Turbulent Boundary Layers

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[JSR **12,** 126-128 (1975)]

IN the Nomenclature, the following changes should be made:

ℓ = mixing length

y_ℓ = characteristic thickness of boundary layer

Equations (1-3) and (5) should read:

$$\ell = ky\ D, \text{ for } 0<y\leq \lambda y_\ell / k \quad (1)$$

$$\ell = \lambda y_\ell, \text{ for } \lambda y_\ell / k < y \quad (2)$$

$$\ell = \mathcal{L}\lambda y_\ell, \text{ for } \mathcal{L}\lambda y_\ell / k < y \quad (3)$$

$$\delta_w^+ = (\rho_w V_{\tau,w} y_\ell)/\mu_w \quad (5)$$

The symbol $y_{\mathcal{L}}$ for the characteristic thickness of the boundary layer should be changed to y_ℓ in the first two paragraphs of the Analysis section.

Reference 6 should read:

[6] McDonald, H., "Mixing Length and Kinematic Eddy Viscosity in a Low Reynolds Number Boundary Layer," Rept. J214453-1, Sept. 1970, Research Lab., United Aircraft Corp., East Hartford, Conn.

Received July 7, 1975.

Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow.